# Tutorial 10

**Resilient Communication Systems**
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RESILIENT COMMUNICATION SYSTEMS

## MIMO Capacity II

## Problem 1

**High and low SNRs:**
Consider a MIMO system with the following channel matrix:

$$\mathbf{H} = \begin{bmatrix} 0.2 & -0.2 & 0.2 \\ 0.1 & 0.4 & -1 \\ 1 & 0.2 & 1 \end{bmatrix}. \tag{1}$$

Assume $\sigma_n^2 = 1$ W, and two extreme signal-to-noise ratios (SNRs) with $P_{\text{TX}} = 30$ dB (high SNR) and $P_{\text{TX}} = -30$ dB (low SNR). Compute approximate values for the MIMO capacity (in bits/s/Hz) at these SNRs (*Hint:* First use svd command in MATLAB, then use the properties of optimal power allocation policy at high and low SNRs, respectively).

## Problem 2

**The maximum diversity gain of $N_T N_R$ is achievable:**
Consider a communication system with transmit and receive diversities using M-QAM as modulation. The transmitter and the receiver have $N_T$ and $N_R$ antennas respectively and the channel is flat-fading. Assume the channel coefficients are zero-mean unit variance complex Gaussian random variables. Show that the maximum diversity gain of $N_T N_R$ is achievable.
*Hint: Follow these steps:*

- The instantaneous symbol error rate of the M-QAM modulation over flat-fading channel can be approximately described as: $\text{SER}(h) = c_A . \exp(-d_A^2 |h|^2 \gamma)$ with $c_A$ being the average number of nearest neighbors, $h$ the flat-fading channel coefficient, $\gamma$ the SNR, and $d_A^2 = d_{\text{min}}^2 / \log_2(M)$, where $d_{\text{min}}$ is the normalized minimum distance and $M$ the number of points in the constellation diagram.
- Devise the optimal communication scheme which achieves the maximum diversity gain. Write down the corresponding exact average SER and show that why it is difficult to obtain the average SER in close-form.
- Find expressions for lower- and the upper-bounds of the average SER and show that both expressions yield the diversity gain of $N_T N_R$. This concludes that the expression for the exact SER has also the diversity gain of $N_T N_R$. (*Hint: Use the properties of the Frobenius norm*).

## Problem 3

**Diversity gain-multiplexing gain trade-off in MIMO systems:**
The multiplexing gain $G_m$ and the diversity gain $G_d$ of a MIMO system can be defined as follows:

$$G_m = \lim_{\gamma \to +\infty} \frac{R(\gamma)}{\log_2(\gamma)}$$
$$G_d = \lim_{\gamma \to +\infty} -\frac{\log(P_e(\gamma))}{\log(\gamma)}$$

where $R$, $P_e$, and $\gamma$ are the achievable rate, probability of error, and SNR, respectively. There is a trade-off between the diversity gain and the multiplexing gain. In [1], it is shown that for a $N_T \times N_R$ MIMO system the optimal diversity gain can be approximately described as follows

$$G_d^{\text{opt}} = (N_T - G_m)(N_R - G_m), \tag{2}$$

where $N_T$, $N_R$, and $G_m$ are the number of transmit antennas, the number of receive antennas, and multiplexing gain, respectively. Now consider a $8 \times 8$ MIMO system with signal-to-noise ratio $\gamma = 10$ dB.

1. Calculate the diversity gain and multiplexing gain for an asymptotic error probability of $P_e = 10^{-4}$. What is the maximum multiplexing gain?
2. What would be the maximum achievable data rate per dimension (bit/s/Hz) using the results of part 1?

David Tse is a distinguished electrical engineer, academic, and researcher, widely recognized for his groundbreaking contributions to wireless communication and information theory. Born in Hong Kong, Tse displayed an early passion for mathematics and engineering. He pursued his undergraduate studies at the University of Waterloo, Canada, where he earned a Bachelor's degree in Systems Design Engineering. He later completed his Ph.D. in Electrical Engineering at the Massachusetts Institute of Technology (MIT) in 1994.

David Tse
Born: 1966

Tse began his academic career as a faculty member at Stanford University, where he delved into research on wireless networks, coding theory, and information theory. His pioneering work has had a profound impact on the development of modern communication systems. In particular, Tse co-authored the book Fundamentals of Wireless Communication, which has become a cornerstone reference in the field. His research introduced innovative concepts in multi-user communication and resource allocation, helping to optimize the performance and efficiency of wireless networks.

A major milestone in Tse's career was his seminal work on opportunistic communication, a technique that exploits variations in channel conditions to enhance data transmission rates. This concept has been instrumental in the design of contemporary cellular systems and wireless standards.

Tse's interdisciplinary approach bridges theory and practice, influencing not only telecommunications but also areas such as genomics and machine learning. His insights into compressed sensing and high-dimensional statistical inference have further expanded the applications of information theory.

Throughout his illustrious career, David Tse has been the recipient of numerous awards and accolades, including the prestigious Claude E. Shannon Award from the IEEE Information Theory Society, recognizing his profound contributions to the field. He has also been elected to the National Academy of Engineering for his innovative research and its transformative impact on wireless communications.

In addition to his academic achievements, Tse is known for his engaging teaching style and dedication to mentoring the next generation of engineers and researchers. His ability to distill complex ideas into accessible concepts has inspired countless students and professionals worldwide.

David Tse's work continues to shape the evolving landscape of wireless communication and information theory, leaving a lasting legacy in the scientific and technological communities. For more information, explore Wikipedia.

## References


[1] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on information theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[2] Tse, David, and Pramod Viswanath. "Fundamentals of wireless communication," Cambridge university press, 2005.

[3] His homepage